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# A simple Stochastic Geometry Model to test a simple adaptive CSMA Protocol: Application for VANETs

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**Abstract**—In wired networks, systems are usually optimized to offer the maximum throughput of point-to-point and generally well identified transmissions. In the first widespread wireless networks such as IEEE 802.11, the model of point-to-point communication still applies; the transmissions are between the wireless nodes and the access point, which usually serves as a gateway to the Internet. Yet this model is no longer valid with more recent wireless systems such as Wireless Sensor Networks (WSNs) and Vehicular Ad Hoc NETWORKS (VANETs). In such networks, communication is between one node and its neighbors and simultaneous transmissions or, in other words spatial reuse, is required to insure good performance. Thus performance is directly linked to the density of successful simultaneous transmissions. Another important remark concerning wireless networks, past and present, is that most of these communications systems use Carrier Sense Multiple Access (CSMA) techniques.

Previous studies, such as [1], show that in CSMA networks, the density of successful transmissions is greatly influenced by the carrier sense detection threshold, which is one of the main parameters of CSMA. In this paper, we use a simple stochastic model for CSMA to experiment an adaptive scheme which tunes the carrier sense threshold to the density of network nodes. This model uses a Matern selection process with a random pattern of nodes distributed as a Poisson Point Process (PPP). Each node in the process receives a random mark and the nodes that have the smallest mark in their neighborhood are elected for transmission, mimicking the election process of CSMA. The analysis in this paper indicates that an adaptive technique can be based on the average waiting times of packets. Alternatively, this technique can also be based on the number of neighboring nodes.

**Keywords**—VANETs, CSMA, spatial performance, stochastic geometry.

## I. INTRODUCTION

Until recently, network performances were evaluated based on the throughput of point-to-point communications. The targeted applications consisted of traffic between a source and destination node. When wireless networks appeared in the late 80s with the development of the IEEE 802.11 technology, this model remained partly valid; wireless nodes sent their packets to the access point and since IEEE 802.11 uses a CSMA technique, only one packet could be transmitted at a time and the paradigm of the transmission with one source and one destination was still present. In more recent wireless networks such as WSNs and VANETs, the density of nodes

and the spatial extension of the network have greatly increased and simultaneous transmissions are required. For example, in VANETs safety packets such as Car Awareness Messages (CAMs) must be sent from each vehicle to the nodes in their neighborhood. Thus with these applications, a legitimate performance metric is the density of successful transmissions from each node to its neighbors. The spatial effect clearly appears because it is not sufficient that one node successfully sends its packets to its neighbors, it is also important that many other nodes in the network are able to send their packets to their neighbors.

To investigate this spatial effect, tools from stochastic geometry such as Poisson Point Processes (PPPs) are very suitable because their coverage is infinite and thus they perfectly capture the spatial effect of networks such as VANETs. Moreover, they can also model random networks. Thus, in the model presented in this paper, we denote the node density by  $\lambda$  and we adopt PPPs to model node locations using the density of successful transmissions as the performance metric.

Whereas spatial Aloha networks are quite easy to model and accurately analyze in PPPs [2] mostly because the pattern of simultaneous transmissions remains a random PPP of intensity  $\lambda p$  where  $p$  is the transmission rate of Aloha, the pattern of simultaneous transmissions in CSMA networks is much more complex to model. Far from being completely accurate, the Matern process describes a selection process of a random PPP which can mimic the CSMA selection rule. To the best of our knowledge there is no other technique available to directly model CSMA<sup>1</sup>. This Matern process can be coupled with a classical Signal over Interference and Noise Ratio (SINR) capture model to analytically compute the density of successful transmissions. This model also allows one to compute an evaluation of the packets' waiting time and thus we will be able to use an analytical model to test an adaptation scheme which tunes the carrier detection threshold to the waiting time of the packets.

The remainder of this paper is organized as follows: Section II briefly reviews related work; Section III describes the model proposed to study CSMA based on the Matern selection process and we derive the four fundamental metrics of our net-

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<sup>1</sup>Other techniques are in fact simulations

work: the transmission probability, the probability of capture, the density of successful transmissions and the average waiting time. In Section IV we propose our carrier sense adaptive technique based on the average waiting time. We report the results of this adaptive technique where several scenarios are studied for 1D and 2D networks. Finally, Section V concludes the paper.

## II. RELATED WORK

The first studies of CSMA followed the seminal paper by Kleinrock [3]. This paper and the other following studies provided a good understanding of the improvement brought by carrier sensing to the standard protocol of the moment which was Aloha. However these studies failed to provide a fine modeling of the backoff techniques required in CSMA systems.

This was however achieved in 2000 with Bianchi's model [4] which represented a major advance in terms of investigating the backoff techniques of CSMA protocols such as IEEE 802.11. However in that paper and in the many that followed in Bianchi's footsteps, the network was still a one-hop network

The first studies which attempted to take spatial reuse into account appeared in the 1980s and concerned only the Aloha protocol [5] [2]. A model for slotted Aloha was later introduced in 1988 by Ghez, Verdu and Schwartz. Their quantitative model was able to capture the situation where several receptions were possible at different locations in the network. In [2] the performance of a network based on this same model was more accurately evaluated. In particular the capture probability and the density of successful transmissions were computed when the distance between the source and the destination was known. These evaluations were possible due to the complete and stateless randomization of the transmitting nodes in Aloha networks. The CSMA interference issue was raised in [6] for a linear network of randomly positioned vehicles, however, the study only considered the nearest interferer case.

The pattern of simultaneous transmissions in CSMA was first evaluated in [7] using the Matern selection process [8]. An other similar process was used in [9] in order to evaluate interferences in CSMA, however, the study did not assess the overall network throughput. The model initially developed in [7] and subsequently enhanced in [10], is extended in this current work.

While the effect of carrier sense detection threshold in CSMA protocols has been studied in [11],[12], the spatial effect of the carrier sense detection threshold was not taken into account. Rather, these studies focused more on the capture probability when all nodes are within the same one-hop range.

A more recent study [13] proposes an adaptive algorithm to adapt the power and improve the spatial reuse which is close to the idea of the present paper but [13] does not propose an analytical model to sustain the algorithm proposed and the adaptation relies on the power whereas, here, we adapt the carrier threshold. The study [14] deals with congestion control techniques in VANETs. In particular the adaptation of the carrier-sense is studied but the article does not focus on the spatial reuse but rather studies the packet delivery ratio versus the distance source destination.

## III. SYSTEM MODEL

In this paper we consider a homogeneous Poisson-Point-Process (PPP)  $\Phi$  extended over a 2D plan ( $\mathcal{S} = \mathbb{R}^2$ ), or along a 1D infinite line  $\mathcal{S} = \mathbb{R}$ . As Vehicular Ad-hoc NETWORKS (VANETs) are generally linear networks, they are usually modeled by 1D networks whereas, Mobile Ad-hoc NETWORKS (MANETs) or Wireless Sensor Networks (WSNs) are modeled by 2D networks. As previously stated, we denote the intensity of the PPP by  $\lambda$ .

We assume that the transmission over a distance  $r$  is affected by a power-law decay  $1/r^\beta$  where  $\beta$  varies between 3 and 6 depending on the propagation conditions and a random fading  $F$ . The power received at distance  $r$  from the source node is thus  $P = \frac{P_0 F}{l(r)}$  and we set  $P_0 = 1$  with  $l(r) = r^\beta$ . We adopt a Rayleigh fading i.e. exponentially distributed with parameter  $\mu$  and thus a mean of  $1/\mu$ .

We also use the well-accepted SIR<sup>2</sup> (Signal-over-Interference-Ratio) with a capture threshold  $T$ . In other words, a successful transmission occurs when the ratio of the received signal divided by the interference (i.e. the other concurrent transmissions) will be greater than  $T$ .

We use a Matern selection process to mimic the CSMA selection process. The principle of the Matern selection process consists in attributing a random mark  $m_i$  to each node  $X_i \in \Phi$  and selecting the node with the lowest mark in its neighborhood. We need to define the neighborhood of a node. We denote by  $F_{i,j}$  the fading for a transmission between  $X_i$  and  $X_j$  and we also introduce the carrier sense threshold  $P_{cs}$ . We define the neighborhood of  $X_i$  as being  $\mathcal{V}(X_i) = \{X_j \in \Phi : F_{i,j}/l(|X_i - X_j|) > P_{cs}\}$ . A node, say  $X_i$  will be selected by the Matern selection process if and only if  $\forall X_j \in \mathcal{V}(X_i) \ m_i < m_j$ , i.e  $X_i$  has the lowest mark  $m_i$  in its neighborhood. It is easy to verify that a selection process is well defined by this property.

In CSMA networks, the selection process is actually performed according to the back-off value which is decremented by the node during idle periods until the transmission of the packet. Thus the node with the lowest back-off time in its neighborhood will be chosen to transmit and the random mark of the Matern selection process can be directly linked to the back-off time. However, in a real CSMA network when a node transmits its packet, the other nodes in its neighborhood have already been eliminated and will no longer be able to eliminate other nodes. This is not the case in the Matern selection process which produces an over-elimination and thus underestimates the density of transmissions. This is illustrated in Figure III.1. Following the Matern selection process, node  $i$  correctly has eliminated node  $o$ ; even so, node  $o$  is still able to eliminate node  $p$ . In contrast, in a real CSMA system, once node  $i$  has eliminated node  $o$ , node  $o$  will no longer be able to eliminate any other neighbor.

We note the medium access indicator of node  $X_i$   $e_i = \mathbb{I}(\forall X_j \in \mathcal{V}(X_i) \ m_i < m_j)$

**Proposition III.1.** *The mean number of neighbors of a node*

<sup>2</sup>We omit thermal noise but it could be easily added, as is explained below. An even more realistic model than the SIR based on a graded SIR model using Shannon's law is possible in our framework though with an increased computational cost.

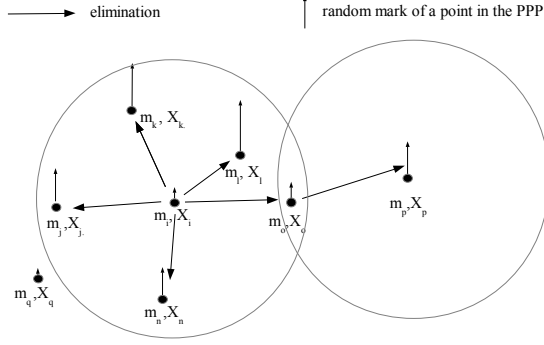


Fig. III.1. Matern CSMA selection process and an example of over-elimination.

is:

$$\mathcal{N} = \lambda \int_{\mathcal{S}} P\{F \geq P_{cs}l(|x|)\}dx.$$

In a 1D network we have :

$$\mathcal{N} = \frac{\lambda\Gamma(1/\beta)}{\beta(P_{cs}\mu)^{1/\beta}}.$$

In a 2D network we have :

$$\mathcal{N} = \frac{2\pi\lambda\Gamma(2/\beta)}{\beta(P_{cs}\mu)^{2/\beta}}.$$

This result is straightforward. Let  $F_j^0$  be the fading between the transmitting node at the origin  $X_i$  and the receiving node  $X_j$ . This is just the application of Slivnyak's theorem and Campbell's formula, see [15], [10]

$$\begin{aligned} \mathcal{N} &= E^0 \left[ \sum_{X_j \in \Phi} \mathbb{1}(F_j^0 l(|X_j - X_i|) \geq P_{cs}) \right] \\ &= \lambda \int_{\mathcal{S}} P\{F \geq P_{cs}l(|x|)\}dx \end{aligned}$$

An immediate computation yields the explicit value of  $\mathcal{N}$  in the 1D and 2D cases.

**Proposition III.2.** The probability  $p$  that a given node  $X_0$  transmits i.e.  $e_0 = 1$  is:

$$p = \mathbf{E}^0[e_0] = \frac{1 - e^{-\mathcal{N}}}{\mathcal{N}}.$$

*Proof:* We compute the probability of a given node at the origin with the mark  $m = t$  being allowed to transmit. Deconditioning on  $t$  provides the result, see [10] for details. ■

If  $p$  is close to 1, then the carrier sense imposes no restriction of transmission. On the other hand, if  $p$  is close to 0, then the carrier imposes a severe restriction on transmission.

**Proposition III.3.** The probability that  $X_0$  transmits given that there is another node  $X_j \in \Phi$  at distance  $r$  is  $p_r$  with

$$p_r = p - e^{-P_{cs}\mu l(r)} \left( \frac{1 - e^{-\mathcal{N}}}{\mathcal{N}^2} - \frac{e^{-\mathcal{N}}}{\mathcal{N}} \right)$$

*Proof:* The proof is the same as that of Proposition III.2. ■

**Proposition III.4.** Let us suppose that  $X_1$  and  $X_2$  are two points in  $\Phi$  such that  $|X_1 - X_2| = r$ . We suppose that node  $X_2$  is retained by the selection process. The probability that  $X_1$  is also retained is:

$$h(r) = \frac{\frac{2}{b(r)-\mathcal{N}} \left( \frac{1-e^{-\mathcal{N}}}{\mathcal{N}} - \frac{1-e^{-b(r)}}{b(r)} \right) (1 - e^{-P_{cs}\mu l(r)})}{\frac{1-e^{-\mathcal{N}}}{\mathcal{N}} - e^{-P_{cs}\mu l(r)} \left( \frac{1-e^{-\mathcal{N}}}{\mathcal{N}^2} - \frac{e^{-\mathcal{N}}}{\mathcal{N}} \right)}$$

with

$$b(r) = 2\mathcal{N} - \lambda \int_{\mathcal{S}} e^{-P_{cs}\mu(l(|x|)+l(|r-x|))} dx.$$

In a 1D network, we have:

$$b(r) = 2\mathcal{N} - \lambda \int_{-\infty}^{\infty} e^{-P_{cs}\mu(l(\tau)+l(|r-\tau|))} d\tau$$

In a 2D network, we have:

$$b(r) = 2\mathcal{N} - \lambda \int_0^{\infty} \int_0^{2\pi} e^{-P_{cs}\mu(l(\tau)+l(\sqrt{\tau^2+r^2-2r\tau\cos(\theta)}))} d\tau d\theta.$$

*Proof:* The proof can be found in [10] for 2D networks. For 1D networks, the formula is a simple adaptation of the 2D case and is left to the reader. ■

**Proposition III.5.** Given the transmission of a packet, we denote by  $p_c(r, P_{cs})$  the probability of successfully receiving this packet at distance  $r$  in a CSMA system (modeled by a Matern selection process with a carrier sense threshold  $P_{cs}$ ) and with a capture threshold  $T$ . We have:

$$p_c(r, P_{cs}) \simeq \exp \left( -\lambda \int_{\mathcal{S}} \frac{h(|x|)}{1 + \frac{l(|x-r|)}{Tl(r)}} dx \right)$$

In a 1D network, we have:

$$p_c(r, P_{cs}) \simeq \exp \left( -\lambda \int_{-\infty}^{\infty} \frac{h(\tau)}{1 + \frac{l(|r-\tau|)}{Tl(r)}} d\tau \right)$$

In a 2D network, we have:

$$p_c(r, P_{cs}) \simeq \exp \left( -\lambda \int_0^{\infty} \int_0^{2\pi} \frac{\tau h(\tau)}{1 + \frac{l(\sqrt{\tau^2+r^2-2r\tau\cos(\theta)})}{Tl(r)}} d\tau d\theta \right)$$

*Proof:* Assuming a packet is transmitted,  $p_c(r, P_{cs})$  denotes the probability of this packet being successfully received at distance  $r$  in a CSMA system using a Matern selection process with carrier sense threshold  $P_{cs}$  and with a capture threshold  $T$ .

The idea is to consider a transmitter at the origin and to evaluate the probability of successful reception by a receiver located at distance  $r$ . We condition the reception of a packet by the presence of another transmitting node at distance  $\tau$ . According to proposition III.4, the density of such nodes is  $\lambda h(\tau)$ . We obtain the result by integrating on  $\tau$ . The details of the proof can be found in [10] for 2D networks. The 1D network case is a simple adaptation of the 2D. ■

It is easy to add a thermal noise  $W$  to the model. The expression of  $p_c(r, P_{cs})$  must then be multiplied by  $\mathcal{L}_W(\mu T l(r))$  where  $\mathcal{L}_W(\cdot)$  is the Laplace Transform of the noise.

**Proposition III.6.** *The spatial density of successful transmissions is thus:*

$$\lambda p p_c(r, P_{cs})$$

*There are 1D and 2D versions of this spatial density and the value of  $p$  and  $p_c(r, P_{cs})$  are chosen accordingly.*

*Proof:* Proposition III.6 is just the exploitation of propositions III.2 and III.5. ■

**Proposition III.7.** *The mean waiting time for a packet (without taking into account the delay in the queue) is  $1/p - 1$  where  $p$  is given by Proposition III.2 and the transmission duration of a packet is one unit.*

*Proof:* The probability of transmitting after waiting for  $i$  slots<sup>3</sup> is  $(1 - p)^i p$  and thus the mean waiting time is

$$\sum_{i=0}^{\infty} p i (1 - p)^i = 1/p - 1.$$

■

#### IV. THE ADAPTIVE CARRIER SENSE THRESHOLD ALGORITHM

Our adaptive protocol will operate in networks modeled by Poisson Point Processes in 1D and 2D geographical areas. We propose to optimize the transmissions for pairs of source-destination nodes at distance  $r$  which is the average distance between a node and its closest neighbor. Thus  $r = 1/\lambda$  and  $r = 1/2\sqrt{\lambda}$  for 1D and 2D networks respectively.

The aim of our adaptive algorithm will be to ensure that  $P_{cs}$  is tuned so that the spatial density of successful transmission as defined in Proposition III.6 is optimized with respect to the spatial density of nodes  $\lambda$  in the Poisson Point Process.

##### A. The underlying idea and results of the algorithm in 1D networks

The idea of our algorithm is based on the fact that when the density of successful transmissions is optimized, the value of  $p$  is always the same when we vary the network density of nodes  $\lambda$ . For instance we verify this conjecture with  $\beta = 2$  and  $T = 10$  and for  $\lambda$  varying between 0.001 and 0.1. In Figure IV.1 we show the optimal value of  $P_{cs}$  to optimize the density of successful transmissions. A simple calculation shows that if  $P_{cs}$  is not carefully selected the density is far below its optimum value. In Figure IV.2 we show that the value of the CSMA transmission probability  $p$  for CSMA, as computed in Proposition III.2, is nearly constant when we vary  $\lambda$  if we optimized the density of successful transmissions with regard to  $P_{cs}$ . Proposition III.7 shows that the network operates optimally when the mean CSMA access delay is a fixed given value:  $D_{target}$  that Section III can precisely evaluate. Thus we use the measurement of the access delay to adapt  $P_{cs}$ .

We propose the following algorithm

The stabilization algorithm continuously updates the value of the average access delay  $D$ . This delay can be simply obtained using a sliding window. Periodically the algorithm compares the current access delay  $D$  with the targeted access delay  $D_{target}$  and updates  $P_{cs}$  accordingly.

---

```

1: while "Stabilization on" do
2:   Estimate the current average access delay  $D$ 
3:   Periodically process the following tests :
4:   if  $D > D_{target}$  then
5:      $P_{cs} = P_{cs} * 2.$ 
6:   end if
7:   if  $D < D_{target}$  then
8:      $P_{cs} = P_{cs}/1.1.$ 
9:   end if
10: end while

```

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If the delay  $D$  is greater than the targeted access delay  $D_{target}$ , the carrier sense threshold  $P_{cs}$  is multiplied by 2 and if  $D$  is smaller than the targeted access delay  $D_{target}$  then  $P_{cs}$  is divided by 1.1.

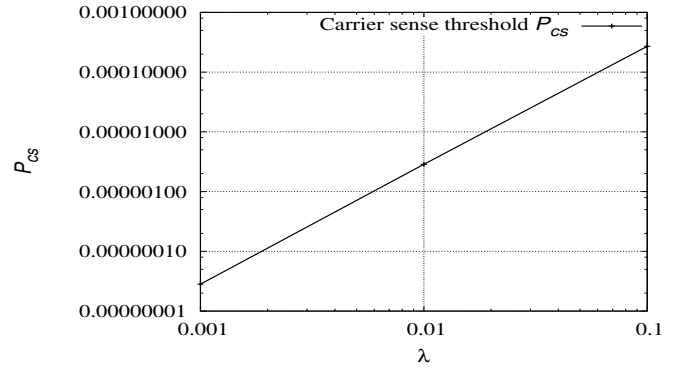


Fig. IV.1. Optimal carrier sense power  $P_{cs}$  versus  $\lambda$  with  $T = 10$ ,  $\beta = 2$ .

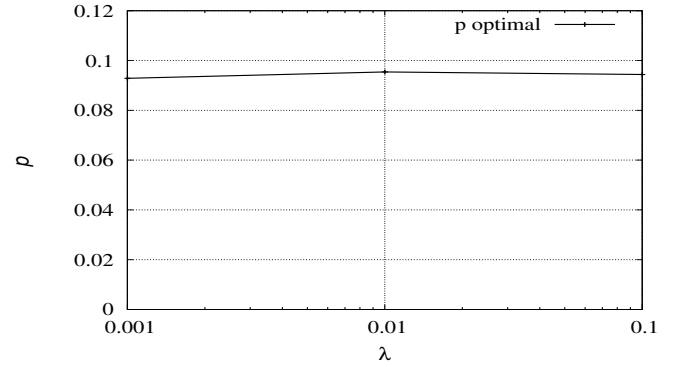


Fig. IV.2. Optimal CSMA transmission  $p$  versus  $\lambda$  with  $T = 10$ ,  $\beta = 2$ .

We test our adaptive algorithm with two examples. In the first example we adopt the following figures  $\mu = 1$ ,  $T = 10$  and  $\beta = 2$ . We also use a random value for  $P_{cs} = 2.8 \cdot 10^{-6}$  and we study how the adaptive algorithm adapts to the actual density  $\lambda = 0.1$ . We assume that every second the adaptive algorithm evaluates  $D$  and updates  $P_{cs}$  accordingly. In Figure IV.3 we show the evolution of the density of successful transmission compared with the optimal density when the carrier threshold is optimized. We observe that in 8 seconds the adaptive algorithm converges in the neighborhood of the optimal value. When the optimal value is reached the algorithm remains close to this optimal.

<sup>3</sup>We assume that the slot is of duration 1 which is supposed to be the duration of the packets.

In Figure IV.4 we continue to study the adaptive algorithm. From time  $t = 1\text{s}$  to  $t = 15\text{s}$  the density of the network is  $\lambda = 0.1$  and then from  $t = 16\text{s}$  to  $t = 30\text{s}$ , the network becomes sparser,  $\lambda = 0.01$  for instance resulting from the end of a traffic jam. At the beginning of the network the carrier threshold is not optimized. We observe the quick adaptation of the algorithm to the actual density of the network.

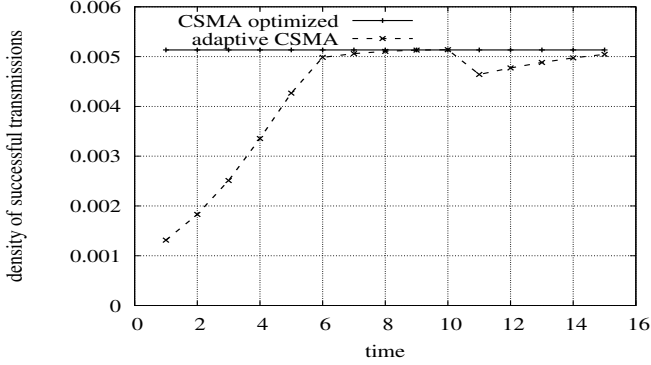


Fig. IV.3. Adaptation of the carrier sense detection threshold to optimize the density of successful transmissions. The initial value of  $P_{cs}$  is for  $\lambda = 0.01$  but the actual value of  $\lambda$  is 0.1. ( $\mu = 1$  and  $T = 10$ ,  $\beta = 2$ ).

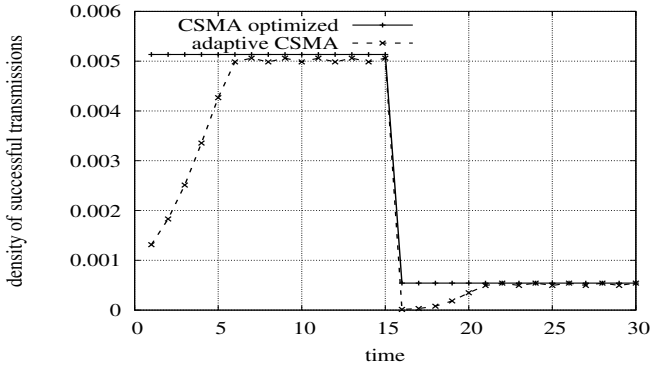


Fig. IV.4. Adaptation of the carrier sense detection threshold to optimize the density of successful transmission. In the beginning of the experimentation  $\lambda = 0.1$  and then at time  $\geq 16$ ,  $\lambda$  becomes 0.01 ( $\mu = 1$  and  $T = 10$ ,  $\beta = 2$ ).

### B. Results in 2D networks

We conduct the same analysis but for 2D networks. In this case we assume that  $\beta = 4$  and we still adopt  $\mu = 1$  and  $T = 10$ . We also vary  $\lambda$  from 0.001 to 0.1. The density of successful transmission is evaluated at the average distance between a node and its nearest neighbor (i.e  $r = 1/(2\sqrt{\lambda})$ ) thus  $r \simeq 1.581\text{ m}$  for  $\lambda = 0.1$ ,  $r = 5\text{ m}$  for  $\lambda = 0.01$  and  $r \simeq 15.81\text{ m}$  for  $\lambda = 0.001$ . Figure IV.5 studies the optimal values of the CSMA transmission probability when we vary  $\lambda$  from 0.001 to 0.1. We observe that this value is around 0.24 irrespectively of the value of  $\lambda$ .

Thus the algorithm described above can be used satisfactorily which is verified that in Figure IV.6. This figure represents the worst case where the carrier sense threshold is set for a small density  $\lambda = 0.001$  whereas the actual density is  $\lambda = 0.1$ . We observe a fast convergence close to the optimal value of

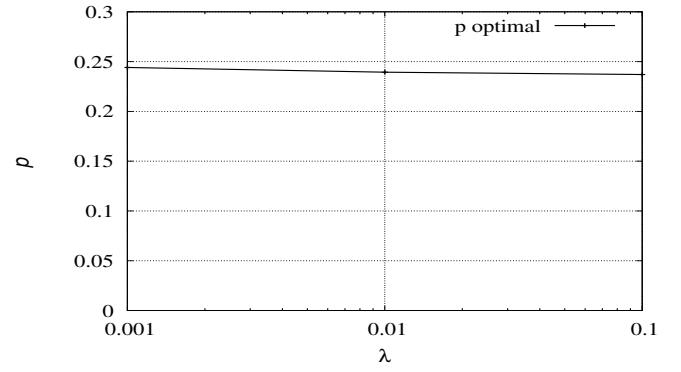


Fig. IV.5. Optimal CSMA transmission  $p$  versus  $\lambda$  with  $T = 10$ ,  $\beta = 4$ .

the density of successful transmissions and then the algorithm maintains the throughput close to this value.

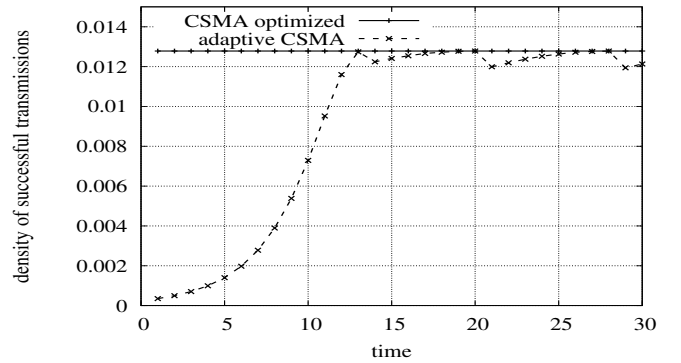


Fig. IV.6. Adaptation of the carrier sense detection threshold to optimize the density of successful transmission in a 2D network. The initial value of  $P_{cs}$  is set to the optimal value for  $\lambda = 0.001$  but the actual value of the network density is  $\lambda$  is 0.1. ( $\mu = 1$  and  $T = 10$ ,  $\beta = 4$ ).

### C. Variant of the adaptive algorithm

We observe that according to Proposition III.2 the CSMA transmission probability  $p$  is directly linked to  $\mathcal{N}$  and thus we can control  $p$  using  $\mathcal{N}$ . This can be implemented monitoring the Cooperative Awareness Messages (CAMs). The adaptive algorithm has to evaluate the mean number of neighbors; CAM packets received below the carrier sense detection threshold are discarded even if they are correctly decoded. Thus a node can determine the number of its neighbors.

- 
- 1: **while** "Stabilization on" **do**
  - 2:   Estimate the current average number of neighbors (i.e nodes from which CAMs are received above the carrier sense threshold)
  - 3:   Periodically process the following tests :
  - 4:   **if**  $\mathcal{N} < \mathcal{N}_{target}$  **then**
  - 5:      $P_{cs} = P_{cs} * 2$ .
  - 6:   **end if**
  - 7:   **if**  $\mathcal{N} > \mathcal{N}_{target}$  **then**
  - 8:      $P_{cs} = P_{cs} / 1.1$ .
  - 9:   **end if**
  - 10: **end while**
-

#### D. Effect of the parameters

We observe that  $p$  and  $p_c(r, P_{cs})$  only depend on the product  $\mu P_{cs}$ . The optimization in  $P_{cs}$  of the density of successful transmissions  $\lambda p p_c(r, P_{cs})$  does not depend on  $\mu$ . Thus the stabilization algorithm can ignore the parameter  $\mu$  to compute the targeted value of  $p$  and then the value of  $D_{target}$ .

In Figure IV.7 we study the optimal value of  $p$  with respect to the capture threshold  $T$ . We observe a clear dependence of  $p$  which requests to know the value of  $T$  to use the adaptive algorithm. This should not really be a problem since this parameter should be known by the designer of the radio transmission system.

In Figure IV.8 we compute the optimal value of  $p$  with respect to the capture threshold  $\beta$ . We observe a clear dependence of  $p$  with  $\beta$  but we note that for  $\beta \geq 3$  the dependence of  $p$  with  $\beta$  tends to be small. Thus an inaccurate evaluation of  $\beta$  will not produce a large error in the evaluation of  $D_{target}$  or  $N_{target}$ , and consequently the adaptive algorithm will continue to provide densities of successful transmissions close to the optimal.

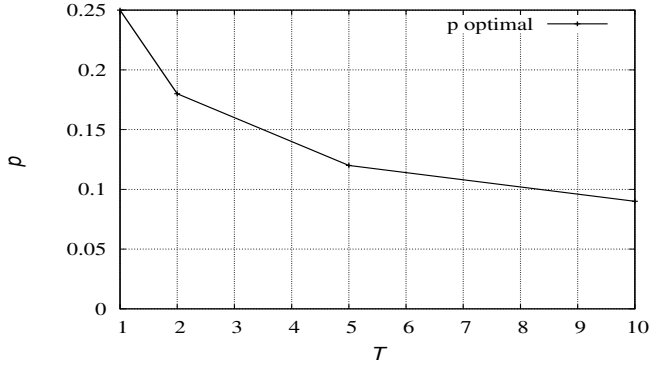


Fig. IV.7. Optimal CSMA transmission  $p$  versus  $T$  with  $\mu = 1$ ,  $\beta = 2$ .

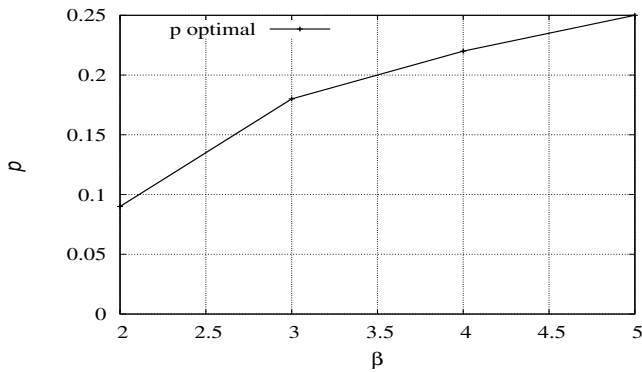


Fig. IV.8. Optimal CSMA transmission  $p$  versus  $\beta$  with  $\mu = 1$ ,  $T = 10$ .

#### V. CONCLUSION

In this paper, we present a simple stochastic model for spatial CSMA in homogeneous Poisson Point Processes. We focus on the transmissions from nodes to their closest neighbors and we observe that the density of successful transmissions strongly depends on the carrier sense threshold  $P_{cs}$  which governs the transmissions in CSMA. The optimal value of  $P_{cs}$  is a function of the density of nodes in the network.

We observe that when the network is optimized (for the density of successful transmission) the value of the optimal value of the transmission probability (denoted by  $p$  in our model) does not depend on  $\lambda$ . Moreover  $p$  is directly linked to the average access delay  $D$  of CSMA. Thus we can design an adaptive transmission algorithm which updates  $P_{cs}$  to reach the optimal value of  $D$ :  $D_{target}$ . We have verified that this stabilization scheme actually adapts  $P_{cs}$  quickly and accurately. We have tested our adaptive algorithm with our analytical model both for 1D and 2D networks. We have successfully studied the influence of the model parameters and found that the results are very encouraging

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